COMPUTATIONAL GEOMECHANICS

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CHAPTER 12

SLOPE STABILITY

In this chapter Bishop's method (Bishop, 1955) for slope stability analysis is presented, together with a simple program. The usual procedure in the analysis of stability of slopes is to calculate the safety factor of various assumed slip surfaces, and then to regard the slip surface having the smallest safety factor as critical. If the safety factor is smaller than 1 the slope is considered to be unstable. In normal conditions the design of such a slope is rejected. In the design of dikes and dams it is usually required that the smallest safety factor is greater than 1, say 1.2 or 1.3.

An unstable slope may be considered acceptable if the unstable condition occurs only in exceptional circumstances, such as in the event of a severe earthquake, coinciding with a high water level. In such cases it may be necessary to predict the deformations that the unstable slope will undergo. If these are small enough the design may still be adequate. In case of a water retaining dam the freeboard will be reduced by the failure, but it is possible that the deformations are so small that the dam keeps its function as a water retaining structure. For this case an estimation of the deformations of an unstable slope will be presented in this chapter, on the basis of a modification of Bishop's method.

12.1 Bishop's method

Bishop's simplified method is based upon a consideration of moment equilibrium of the soil mass above an assumed circular slip surface, see figure 12.1.

The soil mass is subdivided into a number of vertical slices, of width b and height h. The average volumetric weight in a slice is denoted by γ, so that the weight of each slice is γbh. The maximum shear stress acting at the lower boundary of a slice is related to the local cohesion c and the normal effective stress σ' by Coulomb's relation

$$\tau_f = c + \sigma' \tan\phi, \tag{12.1}$$

where ϕ is the angle of internal friction.

It is now assumed that the actual shear stress acting upon the lower boundary of a slice is τ_f/F, where F is a certain constant, the *stability factor*, or *safety factor*. Hence

$$\tau = \frac{1}{F}\,(c + \sigma' \tan\phi). \tag{12.2}$$

It is assumed that the factor F is the same for all slices.

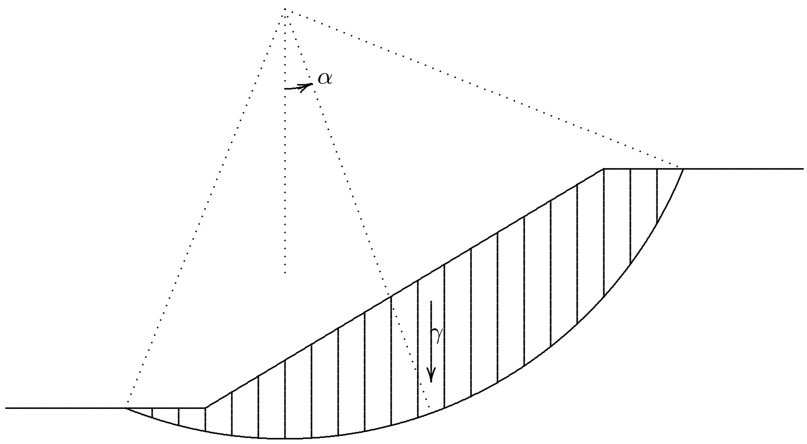

Figure 12.1. Slip circle method.

Equilibrium of moments with respect to the center of the slip circle can be expressed by equating the sum of the moments of the weight of each slice with respect to the center of the circle to the sum of the moments of the shearing forces at the bottom of the slices. Because the horizontal distance from a slice to the center is $R\sin\alpha$ and the area of the bottom section of a slice is $b/\cos\alpha$, this equilibrium condition can be expressed as

$$\sum \gamma hbR\sin\alpha = \sum \frac{\tau bR}{\cos\alpha}. \tag{12.3}$$

If all slices have the same width, it follows from (12.2) and (12.3) that

$$F = \frac{\sum[(c+\sigma'\tan\phi)/\cos\alpha]}{\sum\gamma h\sin\alpha}. \tag{12.4}$$

This formula is the basis of several methods, such as those developed by Fellenius (1927) and Bishop (1955). Because Bishop's method has been validated against solutions for various particular cases and has been used extensively with satisfactory results it will be presented below.

In Bishop's method it is assumed that the forces transmitted between adjacent slices are strictly horizontal. It then follows from the vertical equilibrium of a slice, see figure 12.2, that

$$\gamma h = \sigma' + p + \tau\,\tan\alpha. \tag{12.5}$$

By using the expression (12.2) for the shear stress τ one now obtains

$$\sigma'(1+\frac{\tan\alpha\,\tan\phi}{F}) = \gamma h - p - \frac{c}{F}\,\tan\alpha. \tag{12.6}$$

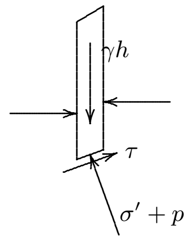

Figure 12.2. Forces on a slice.

Substitution of this expression into equation (12.4) for the stability factor F now gives, finally,

$$F = \frac{\sum \dfrac{c + (\gamma h - p)\tan\phi}{\cos\alpha(1 + \tan\alpha\,\tan\phi/F)}}{\sum \gamma h \sin\alpha}. \tag{12.7}$$

This is the basic formula of Bishop's method. Because the stability factor F also appears in the right hand side of the equation, its value must be determined iteratively, starting with an initial estimate. Experience has shown that the method usually converges very fast, and that the initial estimate can be taken as $F = 1.0$.

It should be noted that in the formula (12.7) the factor γh denotes the total weight of a slice of soil. In an inhomogeneous soil this may be the sum of the weight of a number of sections consisting of different types of soil, from the top of the slice to its bottom. The upper sections of the slice may consist of dry soil, and the lower parts (below the water table) may consist of saturated soil. The shear strength parameters c and ϕ apply to the slip surface, that is the bottom of the slice. In an inhomogeneous soil the values for c and ϕ should of course be taken at the bottom of the slice.

12.2 Koppejan's modification

The maximum shear stress acting at the bottom of a slice is given by

$$\tau_f = \frac{c + (\gamma h - p)\tan\phi}{1 + \tan\alpha\,\tan\phi/F}. \tag{12.8}$$

If $F = 1$ this shear stress becomes infinitely large for $\alpha = \phi - \frac{1}{2}\pi$, because then $\tan\alpha\,\tan\phi = -1$. Such a value for the angle α may occur near the lower end of the slip circle, if the circle is deep, and the friction angle is large. For larger negative values of α the shear stress is negative, which would mean that the shear stress is not acting against the direction of slip. This may lead to unrealistic values for the stability factor, and therefore it has been suggested by A.W. Koppejan of Delft Geotechnics that the value of α to be used in the expression for the shear stress be cut off at $\frac{1}{2}\phi - \frac{1}{4}\pi$, which is one half of the critical value. This is called the modified Bishop method. In most cases the cut-off value is not reached, but it is a refinement that avoids unrealistic values for deep slip circles. This modification has been implemented in the programs used at Delft, and in the program to be presented below.

12.3 Computer program

A computer program that performs the calculations described above is reproduced below. It applies to the situation sketched in figure 12.3. This represents an embankment on an existing homogeneous soil deposit. The groundwater table

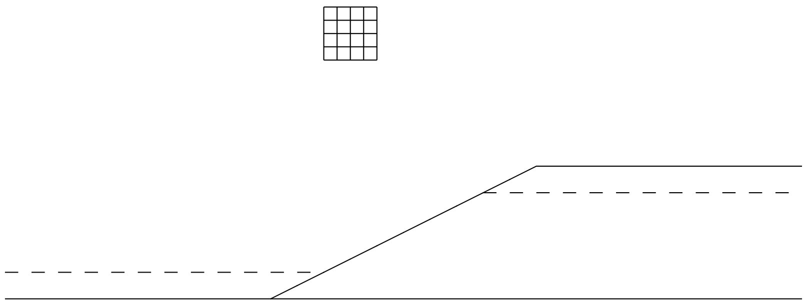

Figure 12.3. Stability of an embankment slope.

inside the embankment may be different from the water table at the downstream slope. The soil properties of the embankment material may be different from those of the subsoil. The slip circles to be investigated are defined by the location of 25 possible centers in a window, and a given lower depth of the slip circles.

```
program slope;
uses crt,graph;
const
 nx=100;mx=4;my=4;
var
 maxx,maxy,graphdriver,graphmode,errorcode:integer;
 xasp,yasp:word;xa,ya,xb,yb,dx,dy,sx,sy:real;
 l,h,h1,h2,gw,gd1,gn1,cc1,phi1,gd2,gn2,cc2,phi2,x1,y1,x2,y2,y0:real;
 s,p:array[1..nx] of real;ff:array[0..mx,0..my] of real;data:text;
procedure title;
 begin
  clrscr;gotoxy(36,1);textbackground(7);textcolor(0);write(' SLOPE ');
  textbackground(0);textcolor(7);writeln;writeln;
 end;
procedure graphinitialize;
 begin
```

```
  graphdriver:=detect;initgraph(graphdriver,graphmode,'');
  errorcode:=graphresult;
  if (errorcode<>grok) then
   begin
    writeln('Error in graphics :',grapherrormsg(errorcode));
    writeln;writeln('Program interrupted.');halt(1);
   end;
  setcolor(7);setbkcolor(0);setlinestyle(0,0,1);
  setfillstyle(11,7);maxx:=getmaxx;maxy:=getmaxy;
  getaspectratio(xasp,yasp);closegraph;
 end;
procedure dots(x1,y1,x2,y2:integer);
 var
  i,x,y,nr:integer;dx,dy,dr,xa,ya:real;
 begin
  xa:=x1;dx:=x2-x1;ya:=y1;dy:=y2-y1;
  if dx<0 then begin xa:=x2;ya:=y2;dx:=-dx;dy:=-dy;end;
  if (x1=x2) and (dy<0) then
   begin
    xa:=x2;ya:=y2;dx:=-dx;dy:=-dy;
   end;
  dr:=(sqrt(dx*dx+4*dy*dy))/4;
  if (dr>0) then begin dx:=dx/dr;dy:=dy/dr;end;
  nr:=trunc(dr);
  for i:=0 to nr do
   begin
    x:=round(xa);y:=round(ya);
    line(x,y,x,y);xa:=xa+dx;ya:=ya+dy;
   end;
 end;
procedure input;
 var
  name:string;
 begin
  title;
  writeln('This is a program for the analysis of the stability');
  writeln('of a slope, using the Bishop method.');writeln;
  write('Name of input datafile ........... ');readln(name);
  assign(data,name);reset(data);readln(data,l,h,h1,h2,gw);
  readln(data,gd1,gn1,cc1,phi1);readln(data,gd2,gn2,cc2,phi2);
  readln(data,x1,y1,x2,y2,y0);
  if (y1<h) then y1:=h;if (y2<y1+0.005) then y2:=y1+0.005;
  if (x2<x1+0.005) then x2:=x1+0.005;
  if (h2>h) then h2:=h;if (h1>h) then h1:=h;
  if (h1<0.0) then h1:=0.0;if (h2<0.0) then h1:=h2;
  close(data);title;
 end;
procedure stability;
 var
  i,j,k,kk,ia,ib,ja,jb:integer;
  xc,yc,xl,yl,xr,yr,r,f,fa,pi,cc,ph,tf,a,b,bb:real;
  dx,x,x3,x4,y,yb,yt,yn,co,si,ta,tb,e,p1,p2,p3:real;
 begin
  clrscr;
  xa:=-l;xb:=2*l;ya:=-h;yb:=h;
  if (x1<xa) then xa:=x1;if (x1>xb) then xb:=x1;
```

```
if (y1<ya) then ya:=y1;if (y1>yb) then yb:=y1;
if (x2<xa) then xa:=x2;if (x2>xb) then xb:=x2;
if (y2<ya) then ya:=y2;if (y2>yb) then yb:=y2;
if (y0<ya) then ya:=y0;if (y0>yb) then yb:=y0;
dx:=xb-xa;dy:=yb-ya;sx:=maxx/dx;sy:=(yasp/xasp)*maxy/dy;
if sy<sx then sx:=sy;sy:=xasp*sx/yasp;
pi:=3.1415926;
for i:=0 to mx do
 begin
  xc:=x1+i*(x2-x1)/mx;
  for j:=0 to my do
   begin
    initgraph(graphdriver,graphmode,'');
    ia:=0;ib:=round(sx*(xb-xa));ja:=maxy-round(-sy*ya);jb:=ja;
    line(ia,ja,ib,jb);
    ia:=round(-sx*xa);ib:=round(sx*(l-xa));
    ja:=jb;jb:=maxy-round(sy*(h-ya));line(ia,ja,ib,jb);
    ia:=ib;ib:=round(sx*(xb-xa));ja:=jb;line(ia,ja,ib,jb);
    for k:=0 to mx do
     begin
      x:=x1+k*(x2-x1)/mx;ia:=round(sx*(x-xa));ib:=ia;
      ja:=maxy-round(sy*(y1-ya));jb:=maxy-round(sy*(y2-ya));
      line(ia,ja,ib,jb);
     end;
    for k:=0 to my do
     begin
      y:=y1+k*(y2-y1)/my;ja:=maxy-round(sy*(y-ya));jb:=ja;
      ia:=round(sx*(x1-xa));ib:=round(sx*(x2-xa));
      line(ia,ja,ib,jb);
     end;
    if (h2<0.0) then
     begin
      ia:=0;ib:=round(sx*(xb-xa));ja:=maxy-round(sy*(h2-ya));jb:=ja;
      dots(ia,ja,ib,jb);
     end
    else
     begin
      ia:=0;ib:=round(sx*(h1*l/h-xa));ja:=maxy-round(sy*(h1-ya));jb:=ja;
      dots(ia,ja,ib,jb);
      ia:=round(sx*(h2*l/h-xa));ib:=round(sx*(xb-xa));
      ja:=maxy-round(sy*(h2-ya));jb:=ja;dots(ia,ja,ib,jb);
     end;
    yc:=y1+j*(y2-y1)/my;r:=yc-y0;
    p1:=1+h*h/(l*l);p2:=-2*xc-2*yc*h/l;p3:=xc*xc+yc*yc-r*r;
    xr:=(-p2+sqrt(p2*p2-4.0*p1*p3))/(2*p1);yr:=xr*h/l;
    if (xr>l) then
     begin
      yr:=h;xr:=xc+sqrt(r*r-(yc-yr)*(yc-yr));
     end;
    xl:=(-p2-sqrt(p2*p2-4.0*p1*p3))/(2*p1);yl:=xl*h/l;
    if (xl<0) then
     begin
      yl:=0.0;xl:=xc-sqrt(r*r-(yc-yl)*(yc-yl));
     end;
    if (yl<h1) then
     begin
```

```
  yl:=h1;xl:=xc-sqrt(r*r-(yc-yl)*(yc-yl));
 end;
dx:=(xr-xl)/nx;
ia:=round(sx*(xc-xa));ja:=maxy-round(sy*(yc-ya));
x:=xl;yb:=yc-sqrt(r*r-(x-xc)*(x-xc));
ib:=round(sx*(x-xa));jb:=maxy-round(sy*(yb-ya));
line(ia,ja,ib,jb);
for k:=1 to nx do
 begin
  x:=xl+(k-0.5)*dx;yb:=yc-sqrt(r*r-(x-xc)*(x-xc));
  ia:=ib;ja:=jb;ib:=round(sx*(x-xa));
  jb:=maxy-round(sy*(yb-ya));line(ia,ja,ib,jb);
  ta:=(x-xc)/(yc-yb);co:=sqrt(1.0/(1.0+ta*ta));si:=co*ta;
  yt:=x*h/l;if (x<0.0) then yt:=0.0;if (x>l) then yt:=h;
  if (h2<0.0) then yn:=h2 else
   begin
    x3:=h1*l/h;x4:=h2*l/h;
    yn:=x*h/l;
    if (x<x3) then yn:=h1;if (x>x4) then yn:=h2;
   end;
  p[k]:=0.0;if (yn>yb) then p[k]:=gw*(yn-yb);
  s[k]:=0.0;if (yb>0.0) then
   begin
    if (yn>yb) then
     begin
      if (yt>yn) then s[k]:=gn1*(yn-yb)+gd1*(yt-yn)
      else
       begin
        if (yt>yb) then s[k]:=gn1*(yt-yb)+gw*(yn-yt)
        else s[k]:=gw*(yn-yb);
       end;
     end
    else if (yt>yb) then s[k]:=gd1*(yt-yb);
   end
  else
   begin
    if (yn>0) then s[k]:=gn2*(0.0-yb) else
     begin
      if (yn>yb) then s[k]:=gn2*(yn-yb)+gd2*(0.0-yn)
      else s[k]:=gd2*(0.0-yb);
     end;
    if ((yt=0.0) and (yn>0)) then s[k]:=s[k]+gw*yn;
    if (yt>0.0) then
     begin
      if (yn>0.0) then
       begin
        if (yt>yn) then s[k]:=s[k]+gn1*yn+gd1*(yt-yn)
        else s[k]:=s[k]+gn1*yt+gw*(yn-yt);
       end
      else s[k]:=s[k]+gd1*yt;
     end;
   end;
 end;
x:=xr;yb:=yc-sqrt(r*r-(x-xc)*(x-xc));
ia:=ib;ja:=jb;ib:=round(sx*(x-xa));
jb:=maxy-round(sy*(yb-ya));line(ia,ja,ib,jb);
```

```
      ia:=round(sx*(xc-xa));ja:=maxy-round(sy*(yc-ya));
      line(ia,ja,ib,jb);
      f:=1.0;e:=1.0;
      while (e>0.001) do
       begin
        a:=0.0;b:=0.0;
        for k:=1 to nx do
         begin
          x:=xl+(k-0.5)*dx;
          yb:=yc-sqrt(r*r-(x-xc)*(x-xc));
          ta:=(x-xc)/(yc-yb);co:=sqrt(1.0/(1.0+ta*ta));si:=co*ta;
          yt:=x*h/l;if (x<0.0) then yt:=0.0;if (x>l) then yt:=h;
          cc:=0.0;ph:=0.0;
          if (yb<0.0) then begin cc:=cc2;ph:=phi2;end
          else if (yb<yt) then begin cc:=cc1;ph:=phi1;end;
          ph:=ph*pi/180;tf:=sin(ph)/cos(ph);
          tb:=sin(0.5*ph-0.25*pi)/cos(0.5*ph-0.25*pi);
          if (ta<tb) then ta:=tb;
          a:=a+(cc+(s[k]-p[k])*tf)/(co*(1+ta*tf/f));
          b:=b+s[k]*si;
         end;
        fa:=a/b;e:=abs(f-fa);f:=fa;
       end;
      ff[i,j]:=f;closegraph;
     end;
   end;
 end;
procedure output;
 var
  i,j:integer;
 begin
  title;
  writeln;writeln('     y    |    Stability factors :');
  writeln('          |');
  for j:=0 to my do
   begin
    write(y2-j*(y2-y1)/my:9:3,' |');
    for i:=0 to mx do write(ff[i,my-j]:9:3);writeln;
   end;
  writeln('    ---------------------------------------------------');
  write('     x    =');for i:=0 to mx do write(x1+i*(x2-x1)/mx:9:3);
  writeln;writeln;
 end;
begin
 graphinitialize;
 input;
 stability;
 output;
end.
```

Program SLOPE.

The program reads data from a datafile, the name of which must be supplied by the user. This datafile must be prepared before, for instance using a word processing program. The meaning of the input parameters is shown in table 12.1. Any

l	Length of the slope
h	Height of the slope
h1	Water level on the left side
h2	Water level on the right side
gw	Volumetric weight of water
gd1	Volumetric weight of soil in embankment, when dry
gn1	Volumetric weight of soil in embankment, when saturated
cc1	Cohesion of soil in embankment
phi1	Friction angle of soil in embankment (in degrees)
gd2	Volumetric weight of soil in subsoil, when dry
gn2	Volumetric weight of soil in subsoil, when saturated
cc2	Cohesion of soil in subsoil
phi2	Friction angle of soil in subsoil (in degrees)
x1	Lower left corner of window of centers
y1	Lower left corner of window of centers
x2	Upper right corner of window of centers
y2	Upper right corner of window of centers
y0	Deepest point of slip circles

Table 12.1. Input parameters program SLOPE.

consistent system of units may be used, for instance meters for length, and kilonewtons for forces. The volumetric weights then are in kN/m^3, and the cohesion is in kN/m^2. The friction angles must be given in degrees.

For each center the program first determines the location of the slip circle, by determining the extreme points, at the right and the left end. This is done by first assuming that the slip circle intersects the slope, and then correcting this assumption if the intersection point is to the right of the upper corner, or to the left of the lower corner. If the water level on the left side is above the soil surface (as in figure 12.3) the slip circle extends to the water surface. The sliding soil mass, above the slip circle, is subdivided into 100 slices. The program determines the lowest point of every slice (`yb`), the location of the soil surface (`yt`), and the location of the water table (`yw`). This then enables to calculate the total stress and the pore pressure at the bottom of the slice. The stability factor F is determined iteratively, until the difference between successive values is less than 0.001.

A sample dataset is shown in table 12.2. This dataset applies to a clay dam

```
10.000   5.000   1.000   4.000  10.000
16.000  20.000  20.000   0.000
16.000  20.000   0.000  30.000
 2.000   9.000   4.000  11.000  -1.000
```

Table 12.2. Dataset SLOPE1.

($c = 20$ kPa, $\phi = 0°$), on a sandy subsoil ($c = 0$ kPa, $\phi = 30°$). During the calculations the slip surfaces considered are shown on the screen, in graphical form. After completion of all calculations output from the program is presented in the form of a table of the stability factors, see table 12.3. In general the calculations

y	Stability factors :				
11.000	1.295	1.275	1.271	1.282	1.308
10.500	1.303	1.277	1.270	1.278	1.302
10.000	1.309	1.279	1.269	1.276	1.299
9.500	1.324	1.288	1.273	1.276	1.296
9.000	1.345	1.300	1.279	1.279	1.296
x =	2.000	2.500	3.000	3.500	4.000

Table 12.3. Output of computer program SLOPE.

should be repeated with a different set of centers until the lowest stability factor is clearly inside the window, and not at one of its boundaries. The deepest point of the slip surfaces should also be varied until the most dangerous slip surface, i.e. the circle with the lowest factor of safety, is obtained.

The program contains a small number of statements to prevent certain inconsistent or impossible situations, such as a groundwater table above the top of the embankment. It does not warn for other errors, however. If a circle does not intersect the soil surface, for instance, the program will accept the data, and then fail during the calculations.

The user may extend the program SLOPE to more general cases, such as embankments of more complicated shape, with variable water levels, and consisting of many soil layers, with variable properties. Programs with such facilities, and with facilities to produce graphical output on various devices, are distributed by various companies and institutes.

12.4 Deformations

The classical slip circle analysis does not give any information about the deformations that will occur when a slope is unstable ($F < 1.000$). In order to obtain a first order estimation of these deformations one may consider the development of the stability factor F when the slope slides along the slip circle, see figure 12.4.

When the soil mass above the slip circle rotates about the center of the slip circle (this is the only form of motion that is kinematically admissible) it can be expected that the driving force is reduced. The moment of the weight with respect to the center is reduced by the movement, which will tend to improve the stability of the soil mass. It should be noted that there is also a negative effect, because at the lower end of the slip circle some soil will loose contact with the base. The analysis of the stability factor after a certain rotation can be performed as follows.

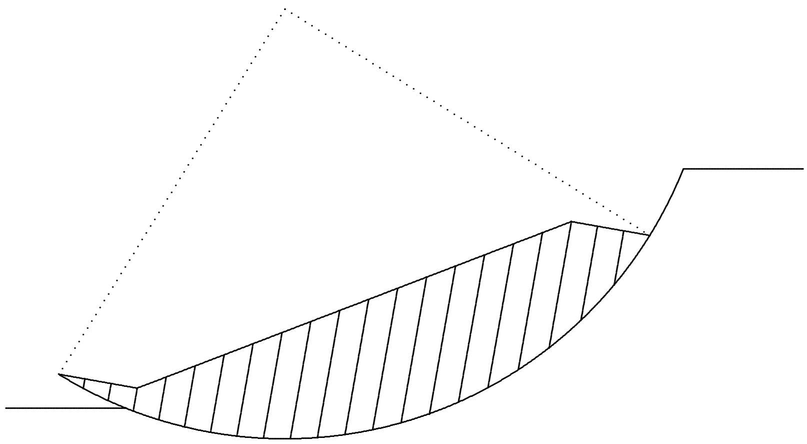

Figure 12.4. Rotation of sliding soil mass.

The general formulas for the displacements due to a rotation over an angle β with respect to the point x_c, y_c can be derived by expressing the coordinates of a point in polar coordinates before and after the rotation. This gives

$$x' = x_c + (x - x_c)\cos\beta + (y - y_c)\sin\beta, \tag{12.9}$$

$$y' = y_c + (y - y_c)\cos\beta - (x - x_c)\sin\beta. \tag{12.10}$$

These formulas can be used to determine the coordinates of the points in the sliding soil mass after rotation.

The basic equation of equilibrium of moments, equation (12.3), now becomes

$$\sum \gamma hb(x'_o - x_c) = \sum \frac{\tau bR}{\cos\alpha}, \tag{12.11}$$

where x'_o is the x-coordinate of the center of mass of a slice (after rotation) and x_c is the $x-$coordinate of the center of the slip circle. Using the angle α to denote the original position of the slice one may write

$$x'_o - x_c = R[\sin\alpha\cos\beta + \frac{(y_o - y_c)}{R}\sin\beta], \tag{12.12}$$

where y_o is the (original) y-coordinate of the center of mass of the slice. The stability factor F can now be expressed as

$$F = \frac{\sum[(c + \sigma'\tan\phi)/\cos\alpha]}{\sum \gamma h[\sin\alpha\cos\beta + (y_o - y_c)\sin\beta/R]}. \tag{12.13}$$

It is assumed, in analogy with the standard Bishop method, that the interaction forces between the slices are horizontal, so that their contribution to the equation

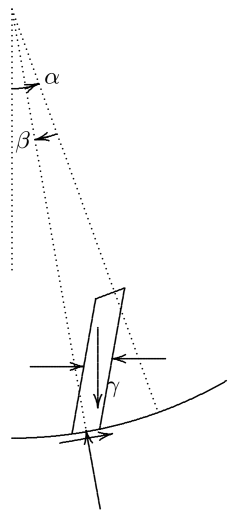

Figure 12.5. Forces on a slice after rotation.

of vertical equilibrium vanishes. The equation of vertical equilibrium of a slice (see figure 12.5) now is

$$\gamma h = (\sigma' + p)\frac{\cos(\alpha-\beta)}{\cos\alpha} + \tau\frac{\sin(\alpha-\beta)}{\cos\alpha}. \tag{12.14}$$

Here it has been assumed that the thickness of the slice is constant during the rotation. Using the expression (12.2) for the shear stress τ one now obtains

$$\begin{aligned}\sigma'[\cos(\alpha-\beta) + \frac{\sin(\alpha-\beta)\ \tan\phi}{F}] = \\ \gamma h\cos\alpha - p\cos(\alpha-\beta) - \frac{c}{F}\ \sin(\alpha-\beta).\end{aligned} \tag{12.15}$$

Substitution of this expression into equation (12.13) leads to the following formula for the stability factor after rotation

$$F = \frac{\sum \dfrac{c\cos(\alpha-\beta) + [\gamma h\cos\alpha - p\cos(\alpha-\beta)]\tan\phi}{\cos\alpha[\cos(\alpha-\beta)+\sin(\alpha-\beta)\ \tan\phi/F]}}{\sum \gamma h[\sin\alpha\cos\beta + (y_o - y_c)\sin\beta/R]}. \tag{12.16}$$

It can easily be seen that for $\beta \rightarrow 0$ this formula reduces to the standard form (12.7).

It should be noted that the summation in the numerator should be extended only over those slices that remain in contact with the subsoil. The summation in the denominator, which expresses the moment of the gravity forces, should be extended over all slices, except in case of a water retaining slope. In the latter case it is assumed that the water level is unaffected by the rotation. In case of a slope above groundwater, as shown in figure 12.4, the summation in the denominator extends over all slices.

In case of a non-homogeneous soil it is not immediately clear what values of the soil strength parameters c and ϕ should be used : those below the slip circle or those above it, or perhaps the average. In the program used to demonstrate the procedure presented in this chapter the values above the slip circle have been chosen, rather arbitrarily.

12.5 Implementation

The extension described above has been implemented into the program STABIL of the Geotechnical Laboratory of the Delft University of Technology. The general idea is that in case of an unstable slope ($F < 1$) a value of the angle β is chosen, such that the stability factor increases to the value 1.000. This must be done iteratively. The final value of the angle β can be used as a measure, or first estimate, of the deformations that can be expected during failure of the slope.

As a first example one may consider an embankment in dry soil, having a slope 1:1, with a rotation of the soil mass above the slip circle over an angle of 45°, see figure 12.6. Irrespective of the soil properties the stability factor now is

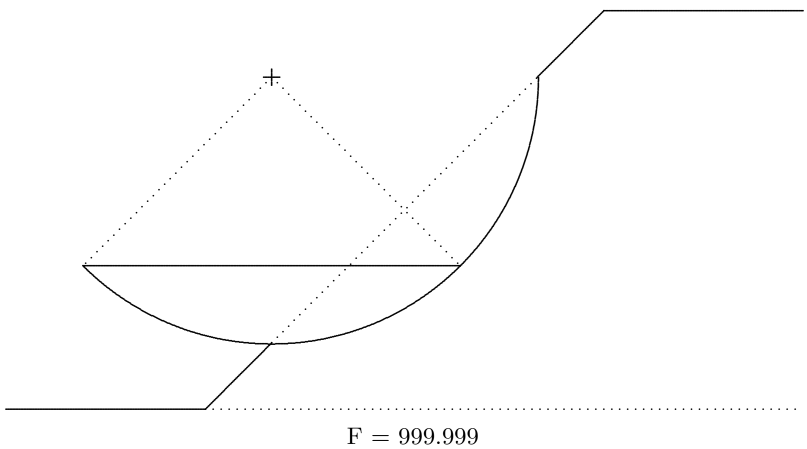

Figure 12.6. Example 1.

found to be practically infinite (as indicated by the maximum value in the program $F = 999.999$), which is in agreement with the fact that the driving moment now vanishes. The procedure leads to an intuitively correct answer in this case.

A second example is shown in figure 12.7. The height of the embankment in this case is 8.00 m, and the slope is 1:1. The soil is purely cohesive ($\phi = 0$), with a cohesion $c = 12$ kPa, a dry volumetric weight of 16 kN/m^3 and a saturated volumetric weight of 20 kN/m^3. The water level is 7 m above base level. The

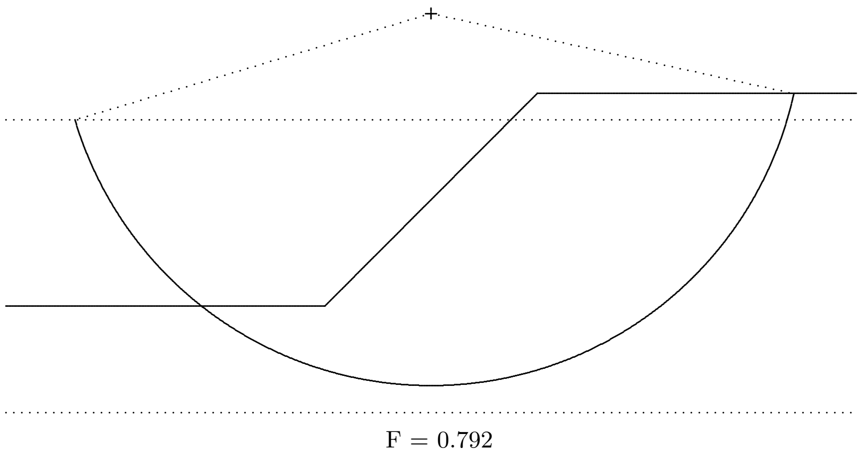

Figure 12.7. Example 2, unstable slope.

stability factor in this case is found to be $F = 0.792$, indicating that the slope is indeed unstable. By trial and error it can be found that by rotating the sliding soil mass over an angle $\beta = 2.85°$ the stability factor is increased to $F = 1.000$, see figure 12.8. Because the radius of the slip circle in this case is 14 m, it now

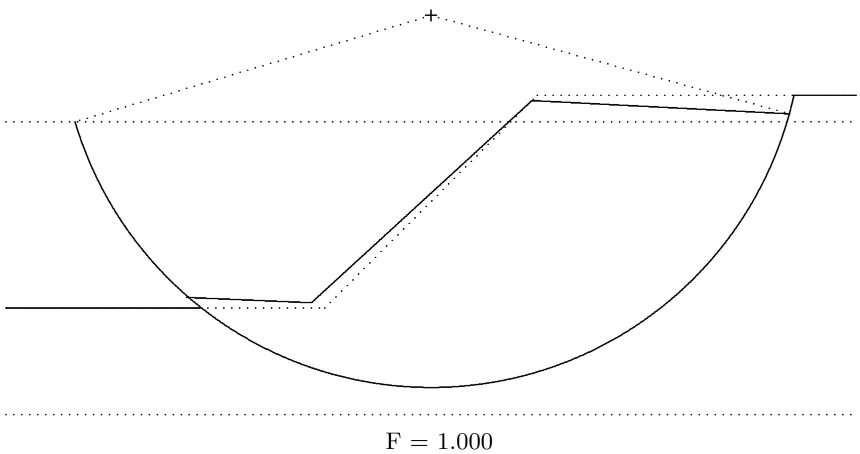

Figure 12.8. Example 2, stable slope after failure.

follows that the maximum vertical displacement is about 0.67 m. This may well be acceptable if the probability of occurrence of this situation, with the parameters used in the calculations, is small. It may be, for instance, that the actual strength of the soil is greater, say $c = 20$ kPa, and that the smaller value of $c = 12$ kPa occurs only after an earthquake, during which the strength of the clay is reduced (This is called degradation of the soil). The embankment appears to be unstable after the earthquake, and will have to be repaired, but the failure is perhaps not catastrophic. Actually, it can be seen from figure 12.8 that after the deformation of the embankment some freeboard remains. In order to verify the outcome of these calculations one may compare them with the results obtained by taking the base level 0.65 m higher, and the level of the embankment 0.65 m lower. In that case the stability factor is found to be $F = 1.000$, which provides some support for the applicability of the deformation analysis presented above.

It can be concluded that the procedure presented in this section leads to reasonable results, and gives a first order estimation of the deformations that can be expected when an unstable slope fails. It should be noted that the method is based on various simplifying assumptions, so that the numerical values should be considered as not more than an indication of the order of magnitude of the real displacements.

Exercises

12.1 Show that the program SLOPE leads to a safety factor close to $F = 1.000$ for an embankment in a homogeneous friction material ($c = 0$) if the slope angle is equal to the friction angle ϕ of the material.

12.2 In a purely cohesive dry material ($\phi = 0$) the maximum height of a vertical cutoff is about 3.83 c/γ, where c is the cohesion and γ is the volumetric weight. Show that the program SLOPE confirms this result.