

# AN INTRODUCTION TO SOIL DYNAMICS

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# PREFACE

This book gives the material for an introductory course on Soil Dynamics, as given for about 10 years at the Delft University of Technology for students of civil engineering, and updated continuously since 1994.

The book presents the basic principles of elastodynamics and the major solutions of problems of interest for geotechnical engineering. For most problems the full analytical derivation of the solution is given, mainly using integral transform methods. These methods are presented briefly in an Appendix. The elastostatic solutions of many problems are also given, as an introduction to the elastodynamic solutions, and as possible limiting states of the corresponding dynamic problems. For a number of problems of elastodynamics of a half space exact solutions are given, in closed form, using methods developed by Pekeris and De Hoop. Some of these basic solutions are derived in full detail, to assist in understanding the beautiful techniques used in deriving them. For many problems the main functions for a computer program to produce numerical data and graphs are given, in C. Some approximations in which the horizontal displacements are disregarded, an approximation suggested by Westergaard and Barends, are also given, because they are much easier to derive, may give a first insight in the response of a foundation, and may be a stepping stone to solving the more difficult complete elastodynamic problems.

The book is directed towards students of engineering, and may be giving more details of the derivations of the solutions than strictly necessary, or than most other books on elastodynamics give, but this may be excused by my own difficulties in studying the subject, and by helping students with similar difficulties.

The book starts with a chapter on the behaviour of the simplest elementary system, a system consisting of a mass, supported by a linear spring and a linear damper. The main purpose of this chapter is to define the basic properties of dynamical systems, for future reference. In this chapter the major forms of damping of importance for soil dynamics problems, viscous damping and hysteretic damping, are defined and their properties are investigated.

Chapters 2 and 3 are devoted to one dimensional problems: wave propagation in piles, and wave propagation in layers due to earthquakes in the underlying layers, as first developed in the 1970's at the University of California, Berkeley. In these chapters the mathematical methods of Laplace and Fourier transforms, characteristics, and separation of variables, are used and compared. Some simple numerical models are also presented.

The next two chapters (4 and 5) deal with the important effect that soils are usually composed of two constituents: solid particles and a fluid, usually water, but perhaps oil, or a mixture of a liquid and gas. Chapter 4 presents the classical theory, due to Terzaghi, of semi-static consolidation, and some elementary solutions. In chapter 5 the extension to the dynamical case is presented, mainly for the one dimensional case, as first presented by De Josselin de Jong and Biot, in 1956. The solution for the propagation of waves in a one dimensional column is presented, leading to the important conclusion that for most problems a practically saturated soil can be considered as a medium in which the solid particles and the fluid move and deform together, which in soil mechanics is usually denoted as a state of undrained deformations. For an elastic solid skeleton this means that the soil behaves as an elastic material with Poisson's ratio close to 0.5.

Chapters 6 and 7 deal with the solution of problems of cylindrical and spherical symmetry. In the chapter on cylindrically symmetric problems the propagation of waves in an

infinite medium introduces Rayleigh's important principle of the radiation condition, which expresses that in an infinite medium no waves can be expected to travel from infinity towards the interior of the body.

Chapters 8 and 9 give the basic theory of the theory of elasticity for static and dynamic problems. Chapter 8 also gives the solution for some of the more difficult problems, involving mixed boundary value conditions. The corresponding dynamic problems still await solution, at least in analytic form. Chapter 9 presents the basics of dynamic problems in elastic continua, including the general properties of the most important types of waves : compression waves, shear waves, Rayleigh waves and Love waves, which appear in other chapters.

Chapter 10, on confined elastodynamics, presents an approximate theory of elastodynamics, in which the horizontal deformations are artificially assumed to vanish, an approximation due to Westergaard and generalized by Barends. This makes it possible to solve a variety of problems by simple means, and resulting in relatively simple solutions. It should be remembered that these are approximate solutions only, and that important features of the complete solutions, such as the generation of Rayleigh waves, are excluded. These approximate solutions are included in the present book because they are so much simpler to derive and to analyze than the full elastodynamic solutions. The full elastodynamic solutions of the problems considered in this chapter are given in chapters 11 – 13.

In soil mechanics the elastostatic solutions for a line load or a distributed load on a half plane are of great importance because they provide basic solutions for the stress distribution in soils due to loads on the surface. In chapters 11 and 12 the solution for two corresponding elastodynamic problems, a line load on a half plane and a strip load on a half plane, are derived. These chapters rely heavily on the theory developed by Cagniard and De Hoop. The solutions for impulse loads, which can be found in many publications, are first given, and then these are used as the basics for the solutions for the stresses in case of a line load constant in time. These solutions should tend towards the well known elastostatic limits, as they indeed do. An important aspect of these solutions is that for large values of time the Rayleigh wave is clearly observed, in agreement with the general wave theory for a half plane. Approximate solutions valid for large values of time, including the Rayleigh waves, are derived for the line load and the strip load. These approximate solutions may be useful as the basis for the analysis of problems with a more general type of loading.

Chapter 13 presents the solution for a point load on an elastic half space, a problem first solved analytically by Pekeris. The solution is derived using integral transforms and an elegant transformation theorem due to Bateman and Pekeris. In this chapter numerical values are obtained using numerical integration of the final integrals.

In chapter 14 some problems of moving loads are considered. Closed form solutions appear to be possible for a moving wave load, and for a moving strip load, assuming that the material possesses some hysteretic damping.

Chapter 15, finally, presents some practical considerations on foundation vibrations. On the basis of solutions derived in earlier chapters approximate solutions are expressed in the form of equivalent springs and dampings.

The text has been prepared using the L<sup>A</sup>T<sub>E</sub>X version (Lamport, 1994) of the program T<sub>E</sub>X (Knuth, 1986). The P<sub>C</sub>T<sub>E</sub>X macros (Wichura, 1987) have been used to prepare the figures. Modern software provides a major impetus to the production of books and papers in facilitating the illustration of complex solutions by numerical and graphical examples. In this book many solutions are accompanied by parts of computer programs that have been used to produce the figures, so that readers can compose their own programs. It is all the more appropriate to acknowledge the effort that must have been made by earlier authors and their associates in producing their publications. A case in point is the paper by Lamb, more than a century ago, with many illustrative figures, for which the computations were made by Mr. Woodall.

A CD-ROM accompanies this book containing programs for waves in piles, propagation of earthquakes in soils, waves in a half space generated by a line load, a point load, a strip load, or a moving load, and the propagation of a shock wave in a saturated elastic porous material. These computer programs are also available from the website <<http://geo.verruijt.net>>. Updates of these programs will be published on this website. Early versions of the book have been published on this website, leading to helpful comments by readers from all over the world.

Many thanks are due to Professor A.T. de Hoop for his many helpful and constructive ideas and comments, and to Dr. C. Cornejo Córdova for several years of joint research. Further comments will be greatly appreciated.

Delft

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# TABLE OF CONTENTS

<b>1. Vibrating Systems</b> .....	1
1.1 Single mass system .....	1
1.2 Characterization of viscosity .....	2
1.3 Free Vibrations .....	2
1.4 Forced Vibrations .....	5
1.5 Equivalent spring and damping .....	8
1.6 Solution by Laplace transform method .....	9
1.7 Hysteretic damping .....	11
<b>2. Waves in Piles</b> .....	14
2.1 One-dimensional wave equation .....	14
2.2 Solution by Laplace transform method .....	15
2.3 Separation of variables .....	17
2.4 Solution by characteristics .....	21
2.5 Reflection and transmission .....	23
2.6 The influence of friction .....	27
2.7 Numerical solution .....	30
2.8 Modeling a pile with friction .....	33
<b>3. Earthquakes in Soft Layers</b> .....	36
3.1 Earthquake parameters .....	36
3.2 Horizontal vibrations .....	38
3.3 Shear waves in a Gibson material .....	41
3.4 Hysteretic damping .....	42
3.5 Numerical solution .....	47
<b>4. Theory of Consolidation</b> .....	51
4.1 Consolidation .....	51
4.2 Conservation of mass .....	53
4.3 Darcy's law .....	56
4.4 Equilibrium equations .....	57
4.5 Drained deformations .....	59
4.6 Undrained deformations .....	59
4.7 Cryer's problem .....	61
4.8 Uncoupled consolidation .....	64
4.9 Terzaghi's problem .....	67

<b>5. Plane Waves in Porous Media</b> .....	71
5.1 Dynamics of porous media .....	71
5.2 The basic differential equations .....	74
5.3 Special cases .....	75
5.4 Analytical solution .....	77
5.5 Numerical solution .....	85
5.6 Conclusion .....	87
<b>6. Cylindrical Waves</b> .....	89
6.1 Static problems .....	89
6.2 Dynamic problems .....	94
6.3 Propagation of a shock wave .....	100
6.4 Radial propagation of shear waves .....	105
<b>7. Spherical Waves</b> .....	108
7.1 Static problems .....	108
7.2 Dynamic problems .....	113
7.3 Propagation of a shock wave .....	117
<b>8. Elastostatics of a Half Space</b> .....	124
8.1 Basic equations of elastostatics .....	124
8.2 Boussinesq problems .....	126
8.3 Fourier transforms .....	130
8.4 Axially symmetric problems .....	133
8.5 Mixed boundary value problems .....	136
8.6 Confined elastostatics .....	143
<b>9. Elastodynamics of a Half Space</b> .....	148
9.1 Basic equations of elastodynamics .....	148
9.2 Compression waves .....	149
9.3 Shear waves .....	149
9.4 Rayleigh waves .....	150
9.5 Love waves .....	154
<b>10. Confined Elastodynamics</b> .....	159
10.1 Line load on half space .....	159
10.2 Line pulse on half space .....	163
10.3 Point load on half space .....	172
10.4 Periodic load on half space .....	174
<b>11. Line Load on Elastic Half Space</b> .....	179
11.1 Line pulse .....	179
11.2 Constant line load .....	208
<b>12. Strip Load on Elastic Half Space</b> .....	233
12.1 The strip pulse problem .....	233
12.2 The strip load problem .....	250

<b>13.</b> Point Load on Elastic Half Space .....	277
13.1 Problem .....	277
13.2 Solution .....	279
<b>14.</b> Moving Loads on Elastic Half Space .....	290
14.1 Moving wave .....	290
14.2 Moving strip load .....	301
<b>15.</b> Foundation Vibrations .....	310
15.1 Foundation response .....	310
15.2 Equivalent spring and damping .....	313
15.3 Soil properties .....	315
15.4 Propagation of vibrations .....	316
15.5 Design criteria .....	316
<b>Appendix A.</b> Integral Transforms .....	319
A.1 Laplace transforms .....	319
A.2 Fourier transforms .....	322
A.3 Hankel transforms .....	329
A.4 De Hoop's inversion method .....	332
<b>Appendix B.</b> Dual Integral Equations .....	336
<b>Appendix C.</b> Bateman-Pekeris Theorem .....	338
References .....	341
Author Index .....	345
Subject Index .....	347